Pulsed Power Engineering: Materials & Passive Components and Devices

U.S. Particle Accelerator School
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Materials & Passive Components and Devices Used in Pulsed Power Engineering

- Materials
  - Conductors
  - Insulators
  - Magnetic material
- Passive components and devices
  - Resistors
  - Capacitors
  - Inductors
  - Transformers
  - Transmission lines
  - Loads
    - Klystrons
    - Beam kickers

Materials

- Generally encounter three types of materials in pulsed power work
  - Conductors
    - Wires & cable
    - Buss bars
    - Shielding
    - Resistors
  - Insulators
    - Cables and bushing
    - Standoffs
    - Capacitors
  - Magnetic
    - Inductors, transformers, and magnetic switches
    - Ferrite and tape-wound

Calculating Resistance

- At low frequency, resistance (R) determined by:
  - $R = \rho\ell/A$ (ohm)
    - Material resistivity, ρ ($\Omega\cdot$cm)
    - Conductor length, ℓ (cm)
    - Conductor cross-sectional area, A (cm^2)
- At high frequency, effective conductor area decreased by “skin effect”
  - Conducted current produces magnetic field
  - Magnetic field induces eddy currents in conductor which oppose/cancel B
  - Eddy currents decay due to material resistance, allow conducted current/magnetic field to penetrate material
  - Skin depth, δ, is the effective conducted current penetration ($B = B_{applied}/e$)
  - $\delta = (2\rho/\mu\omega)^{1/2}$ (meters) for a current of a fixed frequency $\omega=2\pi f$, or $\delta \approx (2t\rho/\mu)^{1/2}$ (meters) for a pulsed current of duration t (sec)
    - Material resistivity, ρ ($\Omega\cdot$m)
    - Material permeability, μ (H/m)
  - $\delta = (6.6/f^{1/2})[(\rho/\rho_c)/(\mu/\mu_o)]^{1/2}$ (cm)
    - Normalized resistivity, (ρ/ρ_c), copper resistivity, $\rho_c = 1.7 \times 10^{-8}$ ($\Omega\cdot$m)
    - Relative permeability, $\mu_r = (\mu/\mu_o)$, permeability of free space, $\mu_o = 4\pi \times 10^{-7}$ (H/m)
  - Litz wire is woven to minimize skin effects

Resistivity of Common Materials

Material	Resistivity @ 20^o C
Aluminum	2.62 μΩ•cm
Be-Cu	5.4 – 11.5 μΩ•cm
Brass (66% Cu, 34% Zn)	3.9 μΩ•cm
Copper (OFHC)	1.72 μΩ•cm
Copper (water pipe)	2.1 μΩ•cm
Graphite (typical)	1.4 mΩ•cm
Gold	2.44 μΩ•cm
Indium	9 μΩ•cm
Iron	9.71 μΩ•cm
Silver	1.62 μΩ•cm
Stainless Steel (typical)	90 μΩ•cm
Steel (0.5% C)	13 – 22 μΩ•cm
Water (purified)	2 X 10^7 Ω•cm (maximum)
Water (tap)	10^4 Ω•cm
Water/$CuSO_4$	25 Ω•cm (minimum)

Insulator Properties

- Insulators are used to isolate and support conductors of differing electric potential
- Typically characterized by two properties
  - Breakdown strength, E_{BD}, electric field which will arc through the material
  - Dielectric constant (relative), $\varepsilon_r = \varepsilon/\varepsilon_o$
- Regularly use solid, liquid, gaseous, and vacuum insulators in pulsed power engineering

Solid Dielectrics

- Can be used as structural elements
- Breakdown through material is irreparable
- Can arc along surface, flashover, typically at $E \approx 0.5\ E_{BD}$
- E_{BD} limited by material imperfections, voids, where corona can occur and gradually degrade material. Therefore E_{BD} decreases with increasing material thickness, as the probability of defects increases.
- $100\ \text{V/mil} < E_{BD} < 1\ \text{kV/mil}$ (typical, >0.1")
  $40\ \text{kV/cm} < E_{BD} < 0.4\ \text{MV/cm}$
- $2 < \varepsilon_r < 10$ (excluding ceramic capacitor materials ~10^3)

Solid Dielectric Properties [1]

Material	Diel. Const. 60 Hz. ε	tan δ	Diel. Const. 1 MHz. ε	tan δ	Diel. Strength* V/mil
Aluminum Oxide	8.80	3.3(-4)	8.80	320	320
Barium Titanate	1250	0.056	1143	0.0105	75
Soda-Borosilicate Glass	4.97	-----	4.84	3.6(-3)	400
Epoxy (Epon RN-48)	4.50	0.05	3.52	0.0142	800
Polycarbonate	3.17	0.009	2.96	0.01	400
Acrylic	4.0	0.016	2.55	0.009	400
Polyimide	3.4	0.002	3.4	0.003	570
Polyvinyl Chloride	3.20	0.0115	2.88	0.016	400
PTFE (Teflon)	2.10	<5(-4)	2.10	<2(-4)	550
Polyethylene	2.26	<2(-4)	2.26	<2(-4)	450
Polypropylene	2.55	<5(-4)	2.55	<5(-4)	650
Paper	3.30	0.010	2.99	0.038	200

*Typical DC values for .10 inch thick samples

[1] From Pulse Power Formulary

$Y(X) \equiv Y \cdot 10^X$

Liquid Dielectrics

- Breakdown strength can be comparable to solids
  - Greatly reduced by introduction of contaminants
- Breakdown damage can be "healed"
  - Arcing may result in conductive (typically carbon) residue
  - Circulation will disburse residue, reduce concentration below threshold
  - Filtration/processing can remove contamination
- Oil is the most common liquid insulator used in pulsed power (you are not a pulsed power engineer until you have been up to your armpits in oil)

Dielectric Oils

- Mineral oils
  - Pulsed power work horse
  - Many trade names (e.g. Sontex, Diala AX), some with additives, electrical properties vary little
  - Polychlorinated Biphenyls (PCB) generally phased out in 60's, but may be present in older systems (new systems usually labeled as "PCB free")
  - Increasing concern about the toxicity/environmental impact of these oils
  - Some plastic and rubber compounds will swell if immersed in mineral oil
  - Hydroscopic (absorbs water)
    - However, it takes a lot of absorbed water to significantly degrade properties
    - Absorbed water can be removed by heating
  - Properties also degraded by air entrainment (avoid centrifugal pumps)
  - For best performance, should be circulated, filtered, de-watered and de-aerated
  - Dielectric constant: ε_r = 2.2 (excellent match to many polymers)
  - Breakdown strength
    - E_{BD} is weakly pulse length dependent, $\alpha\ t^{0.33}$ (see Pulsed Power Formulary)
    - Typical pulsed operation: ~ 100 – 400 kV/cm
    - Typical dc operation: ~40 kV/cm

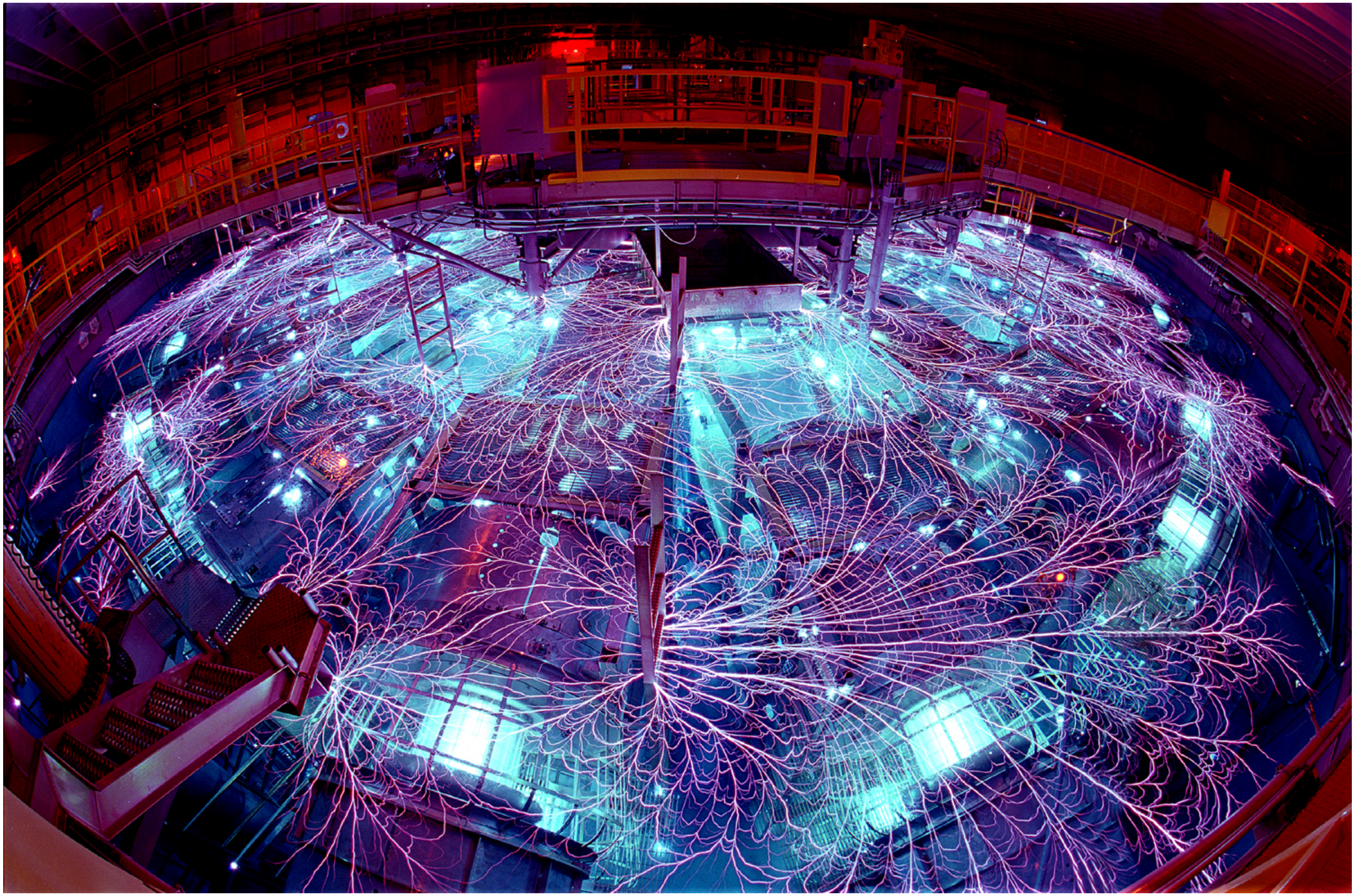
Dielectric Oils (cont.)

- Silicon oils
  - High quality
  - Expensive
- Vegetable oils: castor, rapeseed, canola, etc.
  - Increased usage
  - Low toxicity/environmental impact/high flash point
  - Properties may vary significantly from mineral oils
    - High viscosity, may not be functional at ambient temperatures
    - May support bacterial growth
    - Different dielectric constants; castor ~ 4.5
- Other “oils” used in high value applications (e.g. capacitors)
    - Isopropyl biphenyl
    - benzyltoluene diphenylethane
    - phenyl xylyl ethane
    - tricresyl phosphate
    - ethyl hexyl phthalate

Water as Dielectric

- Water and Ethylene Glycol are often used in PFLs and capacitors
  - High dielectric constants increase pulse length and energy storage
    - Water: $\varepsilon_r = 81$
    - Ethylene Glycol: $\varepsilon_r = 41$
  - Because of low resistivity, can only be used for pulse-charged applications
    - $RC = \rho\varepsilon \sim 2\ \mu s$ maximum for water at 20^o C (However, this can be increased to ~100 ms by mixing ethylene glycol, antifreeze, with the water and chilling the solution to near the freezing temperature.)
  - Breakdown strength
    - E_{BD} is weakly pulse length dependent, $\alpha\ t^{0.33-0.5}$ (see Pulsed Power Formulary)
    - Typical pulsed operation: ~ 50 – 200 kV/cm (~half the strength of oil)

Bulk Breakdown Field Exceeds Surface Flashover Field: SNL Z-machine

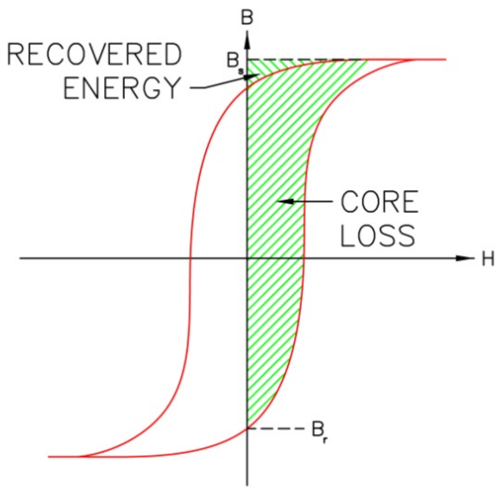


Gaseous Dielectrics

- Breakdowns cause no permanent damage
  - Used for high power switches; spark gaps, thyratrons
  - Produces gas ionization
  - Ion/electron recombination time ~ms (shorter at higher pressure)
- Dielectric constant: $\varepsilon_r = 1$ (low stored energy in stray capacitance)
- Corona (electrical discharge below the breakdown threshold) will ionize gas. This can produce chemical radicals (e.g. O_3) which can degrade system elements.
- Breakdown strength in air:
  - $E_{BD} \approx 25p + 6.7(p/d)^{1/2}$ (kV/cm)
    - Gas pressure, p (atm absolute)
    - Conductor spacing, d (cm)
  - Relative breakdown strength of gases:
    - Air 1.0
    - Nitrogen 1.0
    - SF_6 2.7
    - H_2 0.5
    - 30% SF_6, 70% Air 2.0

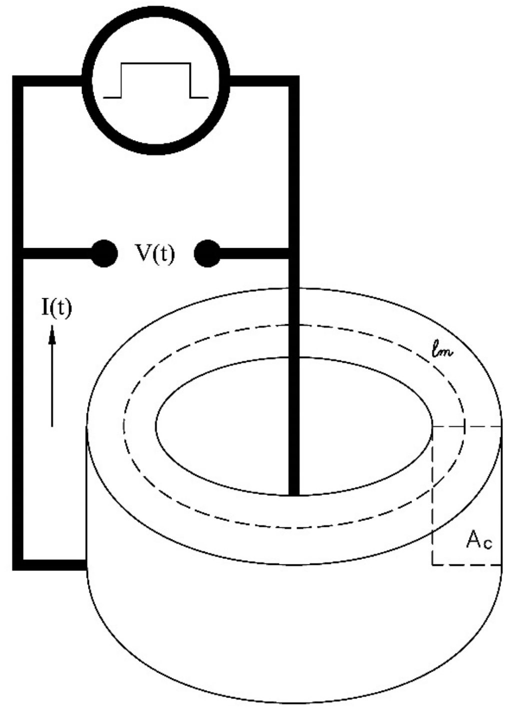
Magnetic Material Properties

- Flux swing, ΔB
  - Change in flux density to saturate ($\mu \rightarrow \mu_o$)
  - Typically remnant flux (H=0), B_r, to saturation flux, B_s: $\Delta B = B_r + B_s$
- Permeability, μ
  - μ(H/m) = B(T)/H(A/m)
  - Permeability of free space, $\mu_o = 4\pi \times 10^{-7}$ H/m
  - Relative permeability, $\mu_r = \mu/\mu_o$ = B(G)/H(Oe)
    - $\mu_r \approx 25{,}000$ for Fe,
    - $\mu_r \approx 400$ for Carbon steel
- Hysteresis loop
  - Plot of B vs H
  - Slope is μ
  - Area is energy



Magnetic Material Properties (cont.)

- Faraday's law
  - $\int \mathbf{B} \cdot d\mathbf{A} = \int V\,dt$
  - $A_c \Delta B = V\tau$
    - Cross sectional area of core, A_c
    - Pulse voltage, V
    - Pulse duration, τ
- Ampere's law
  - $\int \mathbf{H} \cdot d\boldsymbol{\ell} = I$
  - $H = I/\ell_m$
    - Magnetizing current, I
    - Mean magnetic path length, $\ell_m = 2\pi (R_o - R_i) / \ln(R_o / R_i)$ (log mean circumference)

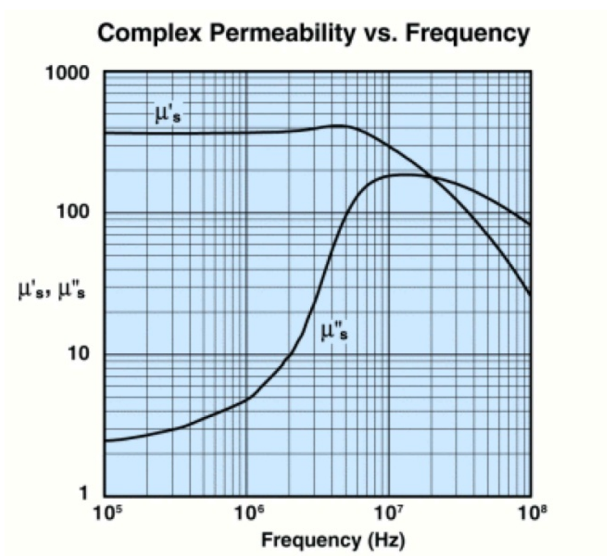


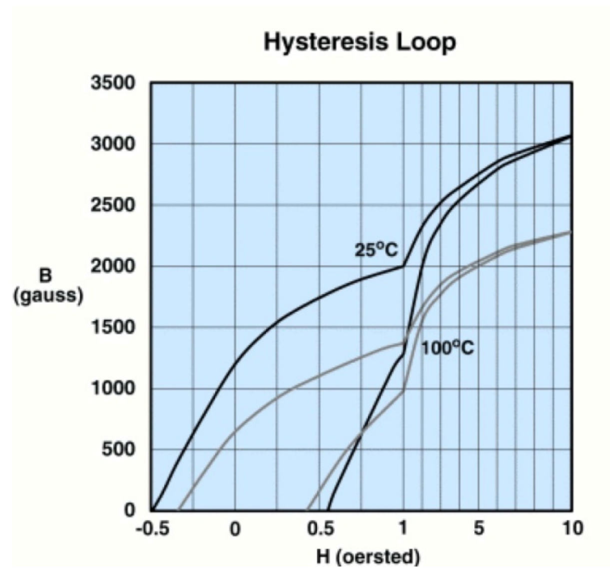
Magnetic Materials

- Two types of material are typically used
  - Ferrimagnetic materials: ferrite cores
    - μ_r: ~500 – 2000 (typical)
    - μ_r approximately constant to >MHz for some formulations
    - ρ: ~10^9 Ω•cm
    - ΔB: ~0.5 T
  - Ferromagnetic materials: "steel" tape-wound cores
    - ρ: ~10^{-5} Ω•cm
      - Eddy currents impede field penetration into material (skin effect)
      - Must be wound from thin (0.001") ribbon interleaved with insulator
      - Insulator does not have magnetic properties, effective area of magnetic material reduced by packing factor, η = insulator thickness/total thickness
    - μ_r: >10^4
    - μ_r strong function of frequency in MHz range for even best materials
    - ΔB: >3 T
- Hysteresis characteristics of any material can be linearized by adding a gap to the core

Ferrite

- Two dominant compositions
  - NiZn
    - Highest frequency response
    - High frequency transformers & chokes, magnetic switching, induction accelerator cores
    - CN20, CMD5005, PE-11B

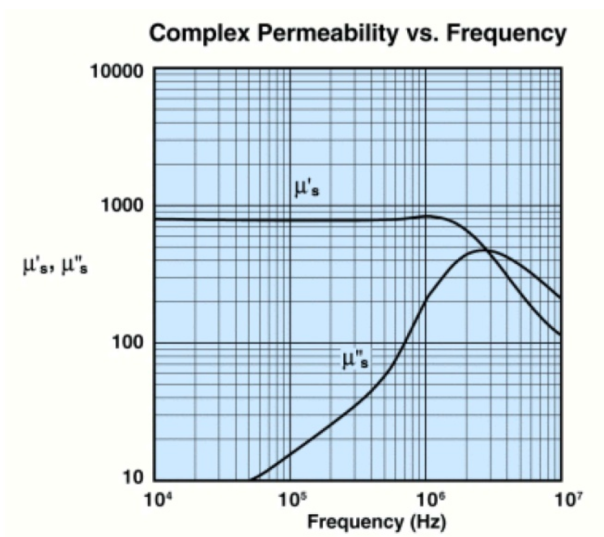


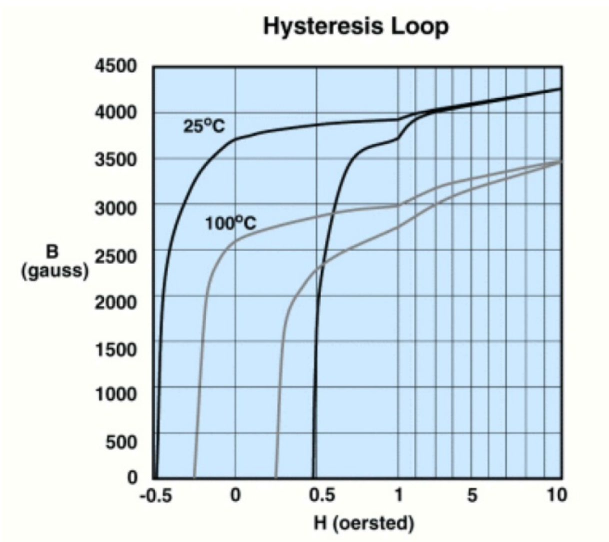


  - Fair-Rite 51
    - Low-loss
    - Modest frequency response (5 MHz)
    - Not square

Ferrite

- Two dominant compositions
  - MnZn
    - Larger ΔB
    - Switch-mode power supply transformers
  - Fair-Rite 85
    - Square loop





Tape Wound Core Materials

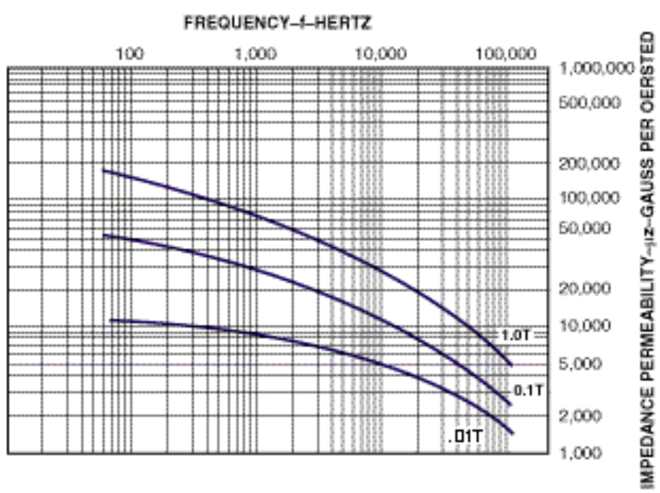
- Crystalline
  - Traditional core material
  - Common formulations: Si-Fe and Ni-Fe
  - Lowest cost
  - Poorest high frequency performance
- Amorphous (Metglas ©)
  - Developed in 70's/80's
  - Iron-based, Ni-Fe-based, and cobalt-based formulations
  - Low loss
  - Higher frequency response
  - Magnetic properties very dependent on annealing
  - Higher costs
- Nano-cyrstalline
  - Iron-based
  - Similar magnetic properties to Metglas
  - Zero magnetostriction

Crystalline Materials

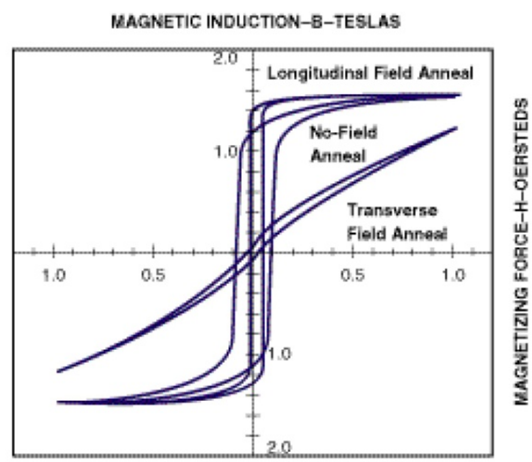
- Si-Fe
  - $\Delta B > 3$ T
  - $\mu_{max} > 25,000$
  - Low frequency applications, 1 – 16 mil thickness
- Ni-Fe
  - $\Delta B \sim 1.5$ T
  - $\mu_{max} > 25,000$ (>100,000 grain oriented material)
  - Thin material, <1 mil, good at higher frequencies, but expensive

Amorphous Materials

- 2605 SA1
  - Most common Fe-based material
  - Modest high frequency response
  - Lowest cost of the amorphous materials
  - $\Delta B \sim 3$ T
  - $\mu_{max} > 100,000$



Typical impedance permeability curves
Longitudinal field anneal



Typical dc hysteresis loops

Amorphous Materials

- 2605CO
  - Fe-based, with cobalt
  - Exceptionally square loop with longitudinal field annealing (lost tech ?)
  - Best material available for high frequency magnetic switching (0.7-mil)
    - $\Delta B = 3.3$ T
    - $\mu_{max} \sim 100{,}000$ (dc)
    - $\mu_{max} \sim 6{,}000$ (1 μs saturation)
    - $\mu_{max} \sim 1{,}000$ (0.1 μs saturation)
- 2714A
  - Co-based
  - Very square, very low loss
  - Best high frequency characteristics
  - $\Delta B = 1$ T
  - $\mu_{max} \sim 500{,}000$ (dc)

Nano-crystaline Materials

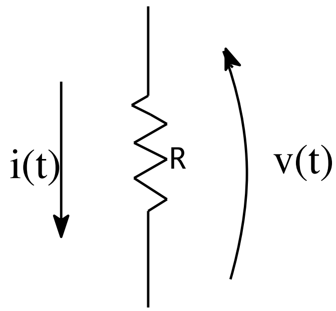
- Similar high frequency permeability and squareness as 2605CO
- $\Delta B \sim 2$ T
- $\mu_{max} \sim 60{,}000$ (dc)
- Major suppliers
  - Hitachi "Finemet"
  - Vacuumschmelze
  - "Russian"
- Hitachi makes excellent cores (including toroids)
  - Well annealed
  - Well constructed (ceramic insulation)

Passive Components and Devices

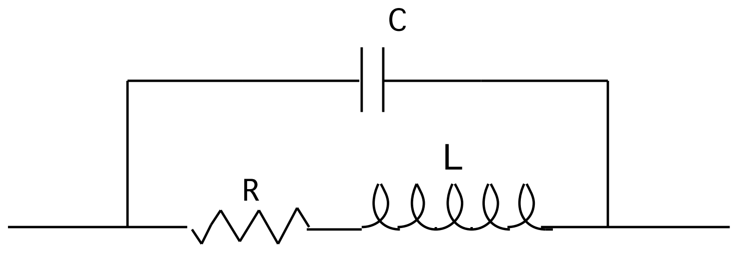
- Resistors
- Capacitors
- Inductors
- Transformers
- Transmission lines
- Loads
  - Klystrons
  - Beam kickers

Resistors

- Resistor behavior



$$v(t) = Ri(t)$$



High-Frequency Equivalent Circuit

Resistor Types

- Film
  - Commonly available
  - Inexpensive
  - Low active material mass → low energy capacity
    - 1W carbon film: ~3 J
    - 1W metal film: ~1 J
  - High voltage film resistors often have a helical pattern → high inductance
    - Alternative, non-inductive serpentine pattern (Caddock)
  - SMD
    - Usually trimmed with an "L-cut", introduces inductance
    - Tend to arc (and fail) at trim, due to V = L dI/dt
- Wire wound
  - Very inductive
  - Large power types (e.g. 225 W) can support large pulsed voltages, but if maintained at high voltage dc, will corona and eventually fail

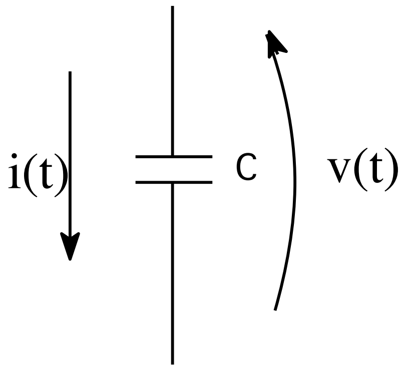
Resistor Types (cont.)

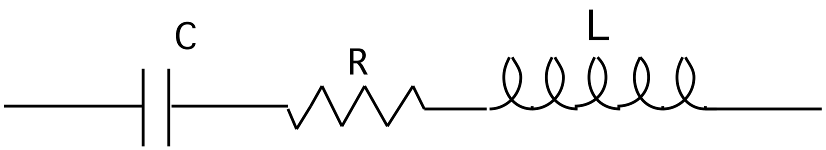
- Composition
  - Large active material mass → large energy handling capacity
  - Carbon Composition
    - 2W “standard” no longer manufactured
    - Voltage and power capacity varies by value
      - 2W: ~80 J, >2 kV repetitive, ~10 kV non-repetitive
  - Ceramic Composition
    - Ohmite OX/OY
    - Even better than carbon comps
    - 2W: ~20 kV non-repetitive
  - Bulk ceramic
    - Stackpole → US Resistor → Kanthal Globar / Carborundum → Cesewid → Kanthal Globar, but also Asian and European manufacturers
    - Vary composition for high voltage, high average power, and high peak power
    - Special coatings for immersion in oil (prevents resistance change)
    - Terminal shape and application critical for long life (corona prevention)
    - Increase average power capacity, ~7X, by flowing water through bore

Resistor Types (cont.)

- Water resistors
  - Typically constructed with insulating tubing (plastic, flexible or rigid, or glass) envelope which contains water with electrodes at each end
  - May be sealed, resistance usually not very stable, or recirculating which can be accurately adjusted
  - Resistivity strongly dependent on water temperature
  - “Salt” is added to provide carriers
    - $CuSO_4$
    - Borax, environmentally benign
    - NaCl
    - KCl
  - Current density on electrodes limited by carrier density (solubility limits)
  - Exceeding $j_{critical}$ (740 mA/cm^2 for $CuSO_4$) → electrode erosion and/or electrolysis
  - Large specific energy deposition → heating → shock wave
- Beam sticks
  - Vacuum diode: $I = \mu V^{1.5}$
  - High power but high cost

- Capacitor behavior





High-Frequency Equivalent Circuit
ESR ≡ parasitic resistance
ESL ≡ parasitic inductance
DF ≡ dissipation factor = RωC

$$q = CV$$

$$i(t) = C\frac{dV(t)}{dt} \quad : \quad \langle i \rangle = C\frac{\Delta V}{\Delta t}$$

$$V = \frac{1}{C}\int i(t)dt$$

Capacitor Types

- Coaxial cable
  - Often acts as capacitor unintentionally
  - $C = \tau/Z$ (transit time/impedance)
- Electrolytic
  - Lossy above ~kHz
  - Low voltage, <kV
  - Energy density: ~1 J/cm^3
  - Limited use in pulsed power, except slow circuits
- Mica
  - High quality
    - Stable
    - Low loss
  - Energy density: ~0.01 J/cm^3
  - Limited distribution above kV, usually made to order

Capacitor Types (cont.)

- Water
  - High energy density ~0.1 J/cm^3 (@200 kV/cm)
  - High voltage, ~MV
  - Due to limited resistivity, only useful in short pulse applications
  - Not commercially available
- Ceramic
  - Available to 50 kV
  - High average current types are available
  - Energy density ~0.025 J/cm^3
  - Capacitance varies with voltage and temperature
  - Stability characterized by "class"
    - I, NPO, COG: most stable
    - II, X7R, Y5P: more variation
    - III: capacitance may decrease 50% at rated voltage

Film Capacitors

- Most commonly used capacitor type for pulsed power applications
- Parameters
  - Voltage: to 100 kV (typically)
  - Current: to 0.25 MA
  - Lifetime: function of
    - Dielectric voltage stress: life α E^x, typically 5 < x < 9
    - Temperature: life is halved for every 10^{o} C increase (polypropylene)
    - Voltage reversal (pulse discharge): dV/dt relative to dielectric relaxation time

Film Capacitor Construction: Dielectric Materials

- Paper (wicks “oil”)
- Polymers
  - Polyester (Mylar®)
  - Polypropylene, High Crystalline Polypropylene (HCPP) best
  - Hazy films wick “oil”
- Oil/fluid (see pages 10 & 11)
- Combinations of the above

Film Capacitor Construction: Conductors

- Foil
  - Aluminum typical (zinc for ac applications)
  - High currents
  - Extended foil (instead of tabs) designs for very high current
- Metalization of dielectric films
  - Lower cost
  - Decrease volume
  - Can be made "self-healing", defects in <2% of film
    - Internal breakdown in film ablates metalization: isolates defect
    - Breakdown energy controlled by controlling metalization
      - Pattern
      - High resistivity metalization, to 0.2 kΩ/□

Film Capacitors Construction: Trade-Offs

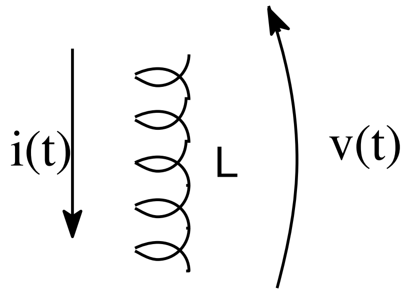
- Film/Foil construction
  - Standard for HV pulse discharge caps
  - Energy density:
    - ~0.02 J/cm^3, typical
    - To ~1 J/cm^3, for high energy density applications (short life)
  - Life
    - Scales as V^7 for a given design
    - >20 year or 10^{10} pulses possible (10^4 - 10^5 typical for high power caps)
- Metalized film construction
  - Higher ESR
    - Lower current capacity
    - Metalization pattern can be tailored to increase current capacity
    - Can be combined with foil to increase current capacity
  - Energy density:
    - 0.1 to 0.3 J/cm^3, typical
  - Life
    - Scales as V^9 for a given design
    - >20 year or 10^{10} pulses possible
    - Self-healing: C drops as metalization erodes, $\Delta C = 5\%$ is end-of-life

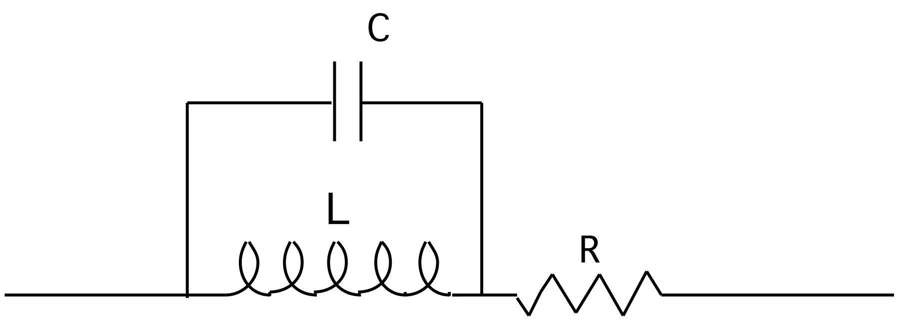
Component Manufacturer Websites

- Capacitors
  - NWL: http://www.nwl.com/contents/view/12
  - Cornell-Dubilier: http://www.cde.com/capacitors
  - TDK: http://www.component.tdk.com/product-portal.php
  - Electronic Concepts: http://www.ecicaps.com/
  - Novacap: http://www.knowlescapacitors.com/novacap
  - CSI: http://www.csicapacitors.com/
  - GA/Maxwell: http://www.ga.com/capacitors
  - WIMA: http://www.wima.com/en_index.php
- Resistors
  - EBG Resistors: http://ebgusa.com/
  - RCD Components: http://www.rcd-comp.com/rcd/index.htm
  - HVR Advanced Power Components: http://www.hvrapc.com/
  - International Resistive Co.: http://www.ttelectronicsresistors.com/
  - Kanthal Globar: http://www.globar.com/
  - Caddock Resistors: http://www.caddock.com/
  - Ohmite: http://www.ohmite.com/

Inductors

- Inductor behavior





High-Frequency Equivalent Circuit

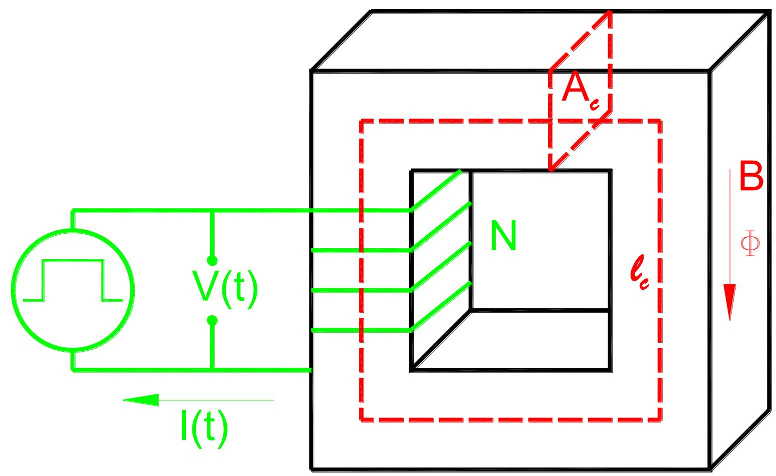
Henry's Law

$$V(t) = L\frac{di(t)}{dt} \quad : \quad \langle V \rangle = L\frac{\Delta i}{\Delta t}$$

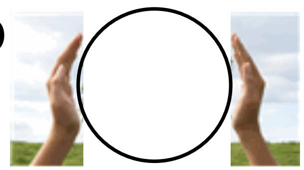
$$i(t) = \frac{1}{L}\int V(t)dt$$

Magnetic Flux & Inductance

- Ampere's Law
  - $\oint H \cdot dl = \oiint J \cdot dA$
  - $Hl_c = NI$
- By definition
  - $B = \mu H = \Phi/Ac$
- Therefore
  - $\Phi = \mu A_c H = \mu Ac\,\mathrm{NI}/l_c$
- Faraday's Law
  - $\oint E \cdot dl = -\frac{d}{dt}\int B \cdot dA$
  - $V/N = -\frac{d}{dt}\Phi$
- Henry's Law
  - $V = L\frac{dI}{dt} = N\frac{d}{dt}\Phi = \frac{dI}{dt}\mu\, Ac\,\mathrm{N}^2/l_c$
  - $L = \mu\, Ac\,\mathrm{N}^2/l_c$

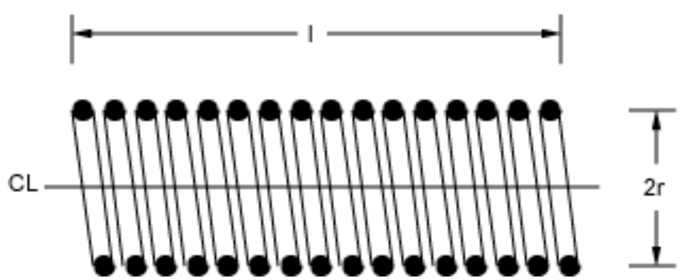


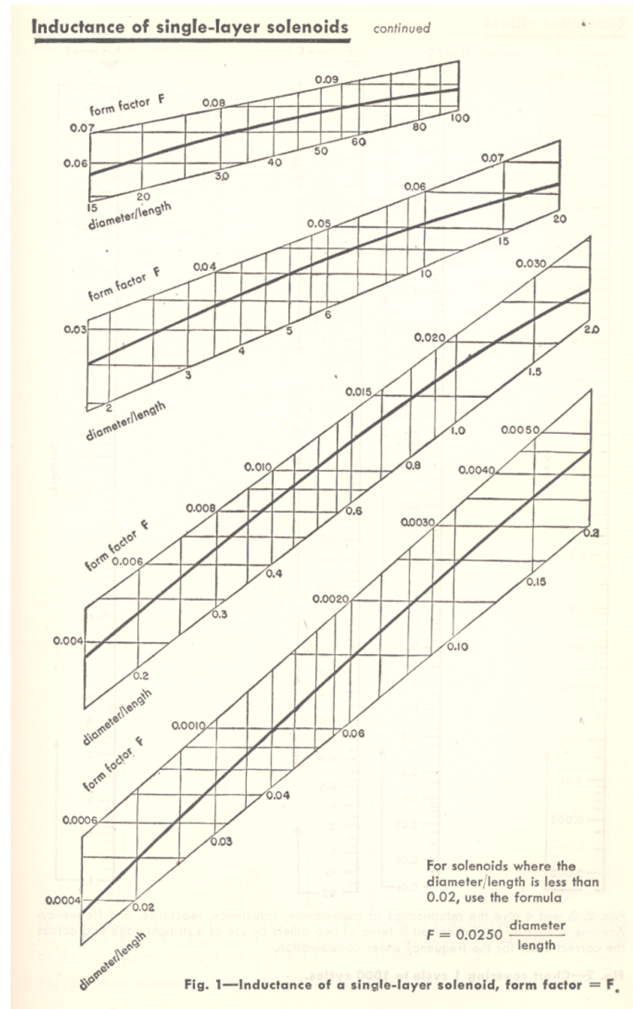
Inductor Types

- Coaxial cable
  - Often acts as inductor unintentionally
  - $L = \tau Z$ (transit time • impedance)
- Current loop
  - 10 μH = 
  - $L = N^2 (a/100) [7.353 \log(16a/d) – 6.386]$ (μH)
    - N turns
    - On radius of a (inch)
    - Of d (inch) diameter conductor, (a/d > 2.5)

Inductor Types

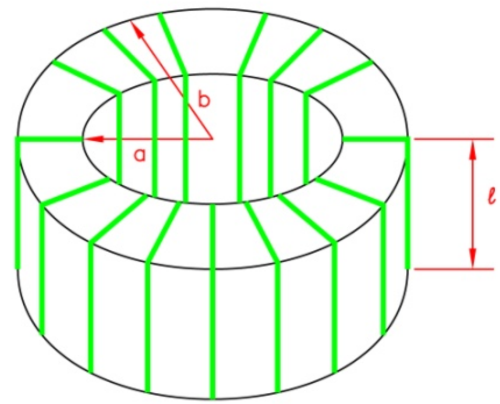
- Solenoid
  - Ideal: $L = N^2 \mu \pi r^2 / \ell$ (SI)
  - Typical: $L = N^2 [r^2/(9r + 10\ell)$ (μH)
  - Generally: $L = F N^2 d$ (μH)
    - Single-layer solenoid
    - N turns
    - Radius: r
    - Diameter: d
    - Length: ℓ

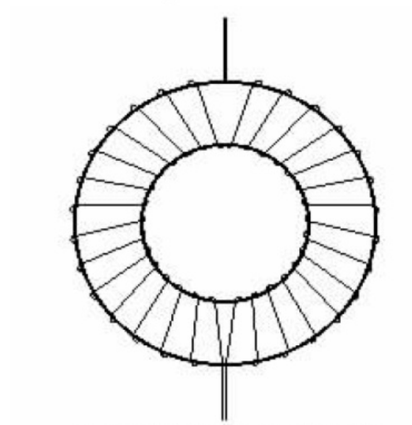




Inductor Types

- Toroid
  - Closed field lines, minimize interaction with adjoining components
  - $L = (N^2 \mu \ell / 2\pi) \ln(b/a)$ (H)
    - N turns
    - Toroid outer radius, b (m)
    - Toroid inner radius, a (m)
    - Toroid length/thickness, ℓ (m)
  - Double ended for HV
    - Better voltage grading around toroid





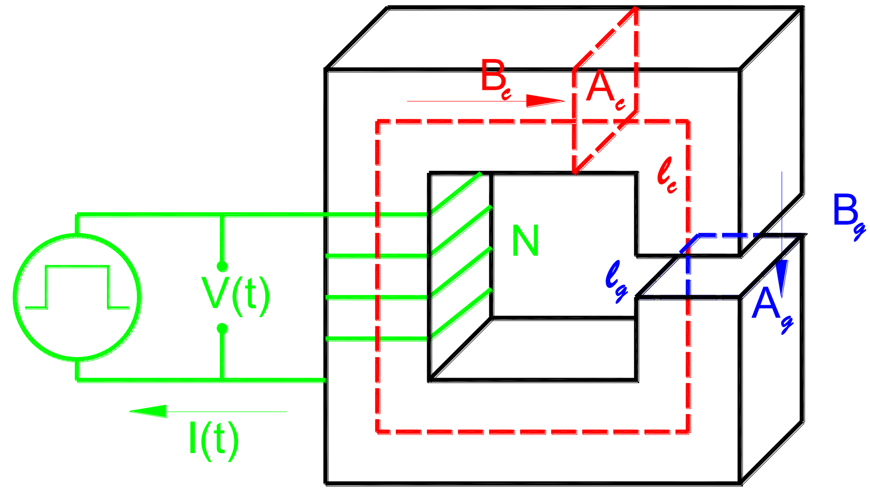
DOUBLE ENDED TORUS

Increasing Inductance with a High Permeability Core

- Air core: $\mu = \mu_o$
  - Constant, independent of frequency and current (subject to parasitic effects)
  - Low permeability
- "Cored" (i.e. filled with magnetic material): $\mu = \mu_o \mu_r$
  - μ_r as high as $>10^5$
  - $\mu = f(\omega, I, \text{temperature})$
  - $V \tau$ constraint
- Compromise: gapped core

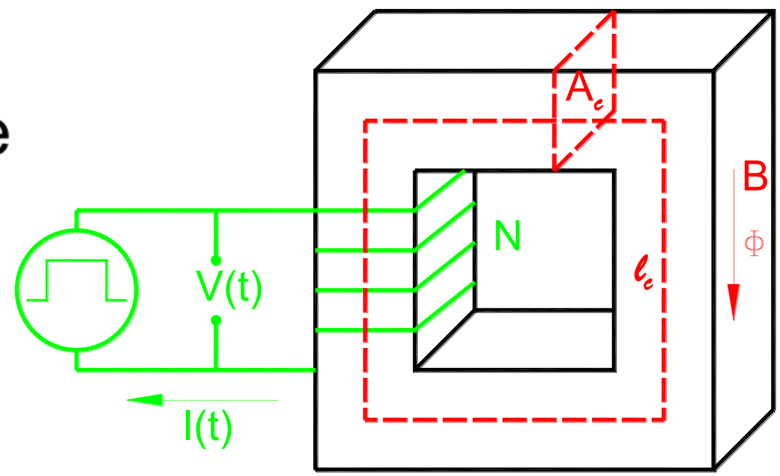
Gapped Core Inductor

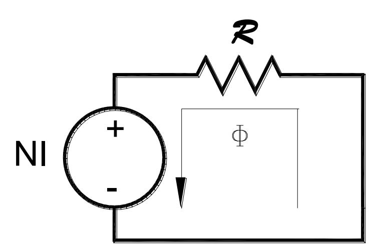
- From Ampere's law
  - $NI = H_c l_c + H_g l_g$
- Flux continuity
  - $\Phi = B_c A_c = B_g A_g$
- Since $B = \mu H$
  - $H_c = \frac{\Phi}{\mu_c A_c}$, $H_g = \frac{\Phi}{\mu_g A_g}$
  - $NI = \Phi \left[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g} \right]$
- Since $N\Phi = LI$
  - $L = \frac{N^2}{\left[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g} \right]}$



Magnetic Circuit

- Magnetic flux, Φ, flows along a continuous path around core
- Flux is driven by magneto-motive force, NI
- Flux is opposed by the circuit reluctance, $\mathcal{R} = l/\mu A$
- Analogy to electrical circuit
  - $\Phi \leftrightarrow I$
  - $NI \leftrightarrow V$
  - $\mathcal{R} \leftrightarrow R$
    - Series & parallel similarly
  - $NI = \Phi\mathcal{R} = \Phi l/\mu A$





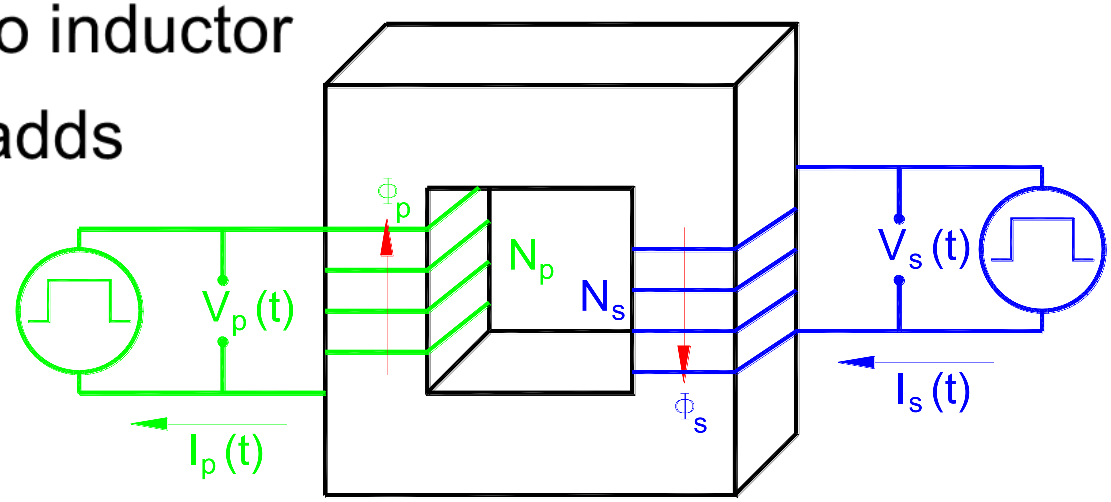
Impacts of Gapping Inductor Core

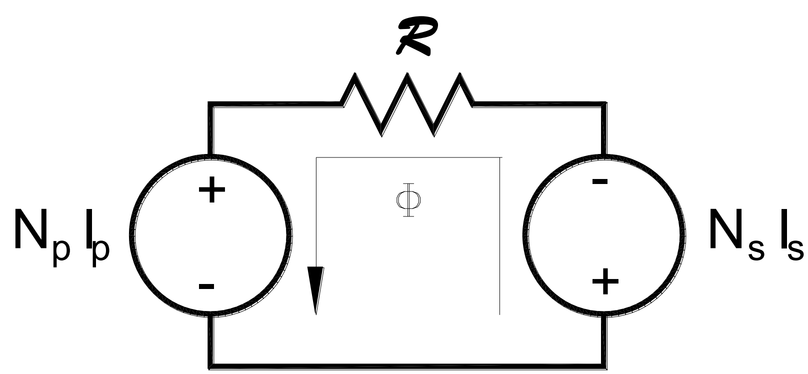
- Total reluctance, $\mathcal{R} = \dfrac{l_c}{\mu_c A_c} + \dfrac{l_g}{\mu_g A_g}$
- Compare: $\dfrac{l_c}{\mu_c A_c}$ to $\dfrac{l_g}{\mu_g A_g}$
  - $Ac \approx Ag$, $\mu_c/\mu_g \sim 10^4$, $l_c/l_g \sim 10^2$
  - $\dfrac{l_c}{\mu_c A_c} << \dfrac{l_g}{\mu_g A_g}$
- Therefore
  - $\mathcal{R} \approx \dfrac{l_g}{\mu_g A_g}$
  - $L \approx \dfrac{\mu_g A_g N^2}{l_g} = N^2/\mathcal{R}$
  - Inductance is decreased when the core is gapped
  - Inductance is virtually independent of μ_c
  - Inductor can store much more energy (energy mostly in gap)

Inductor Considerations

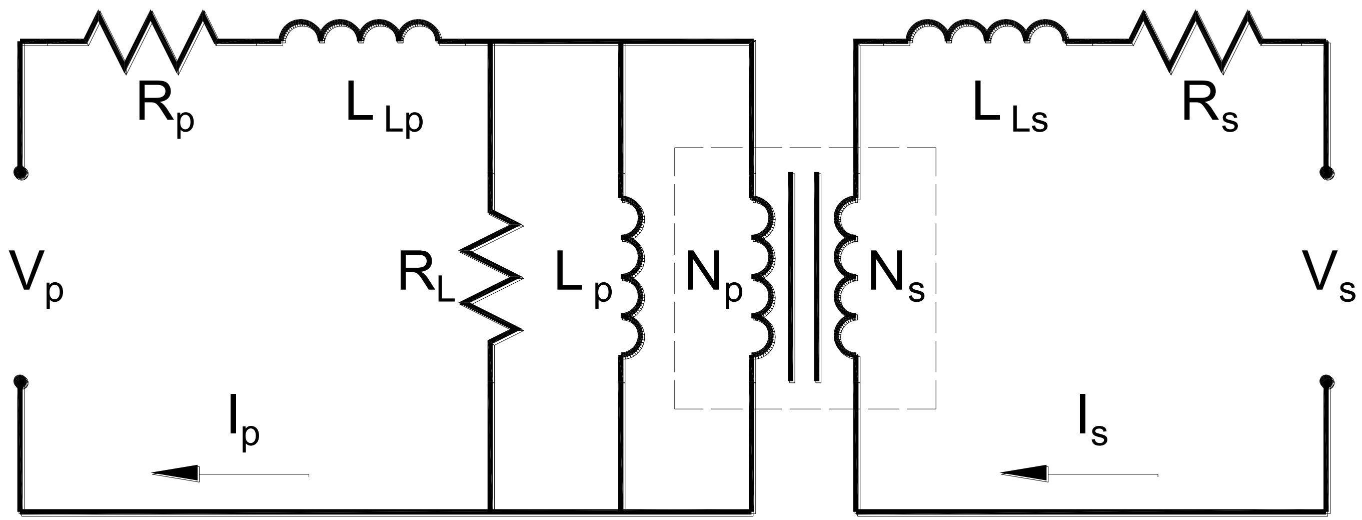
- Quality factor
  - $Q = \omega L / ESR$ (inverse of capacitor dissipation factor)
  - Energy loss per cycle / total stored energy
- Commercial inductors are generally made “to order”
  - Magna Stangenes (Stangenes Industries)

- Add a second winding to inductor
- By super-position, flux adds
  - $\Phi\mathcal{R} = N_p I_p + N_s I_s$
- "Ideal" transformer
  - $\mu_c \rightarrow \infty$
  - $\mathcal{R} \rightarrow 0$
  - $N_p I_p + N_s I_s = 0$
- Faraday's law
  - $-\frac{d}{dt}\Phi = \left(\frac{V}{N}\right)_p = \left(\frac{V}{N}\right)_s$
  - $\frac{V_p}{N_p} = \frac{V_s}{N_s}$





Transformer Model



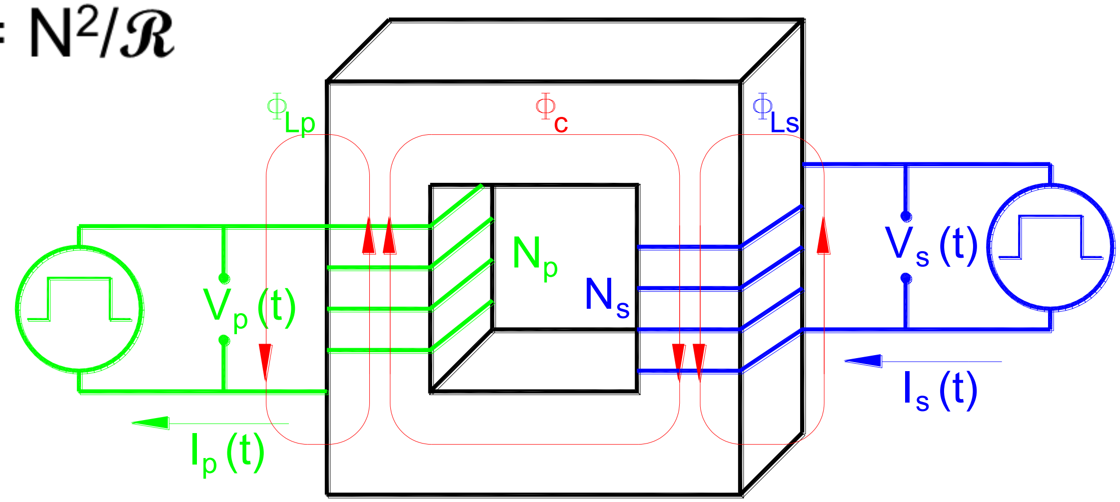
Ideal transformer

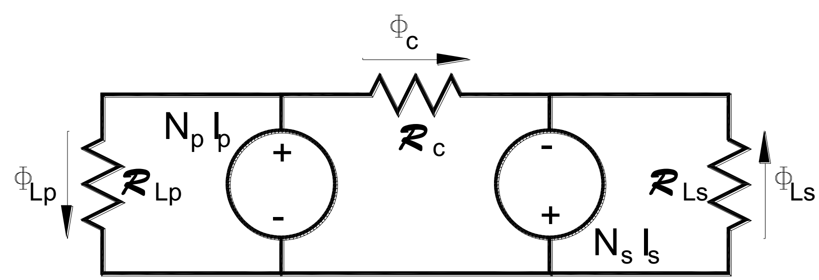
- Ideal transformer identities
  - $V_s/V_p = N$
  - $I_p/I_s = N$
  - $Z_s/Z_p = N^2$

- Loss terms
  - Primary winding: R_p
  - Secondary winding: R_s
  - Equivalent core loss: R_L
- Inductance terms
  - Primary leakage: L_{Lp}
  - Secondary leakage: L_{Ls}
  - Primary (magnetization): L_p
- Winding turns
  - Primary: N_p
  - Secondary: N_s
  - Ratio: $N = N_s/N_p$

Apply Circuit Analogy to Transformer Model

- Noting: $\Phi = NI/\mathcal{R}$ and $L = N^2/\mathcal{R}$
  - $\Phi_{Lp} = \frac{N_p I_p}{R_{Lp}}$
  - $\Phi_{Ls} = \frac{N_s I_s}{R_{Ls}}$
  - $\Phi_c = \frac{N_p I_p + N_s I_s}{R_c}$
  - $L_{Lp} = \frac{N_p^{\ 2}}{R_{Lp}}$
  - $L_{Ls} = \frac{N_s^{\ 2}}{R_{Ls}}$
  - $L_c = \frac{N_p^{\ 2}}{R_c}$ (primary referenced)



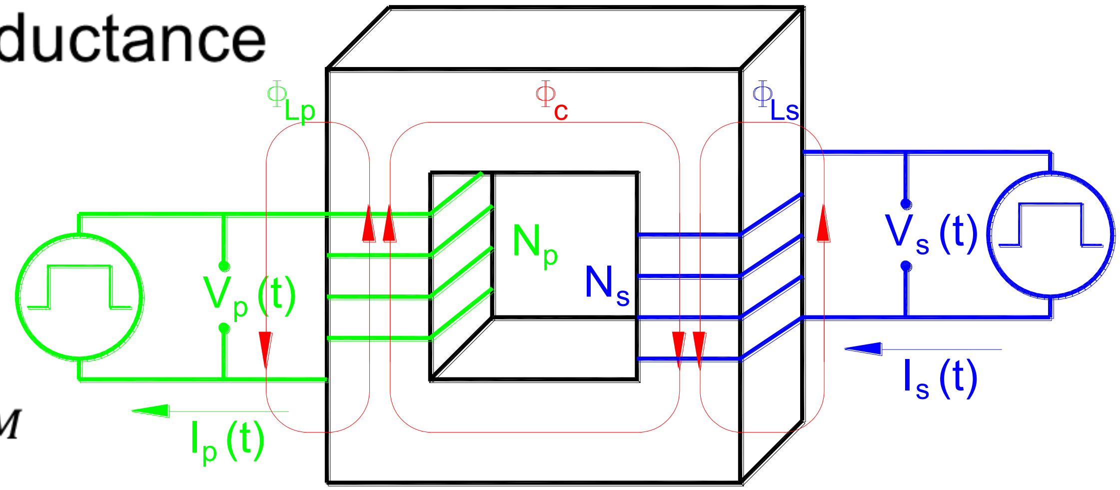


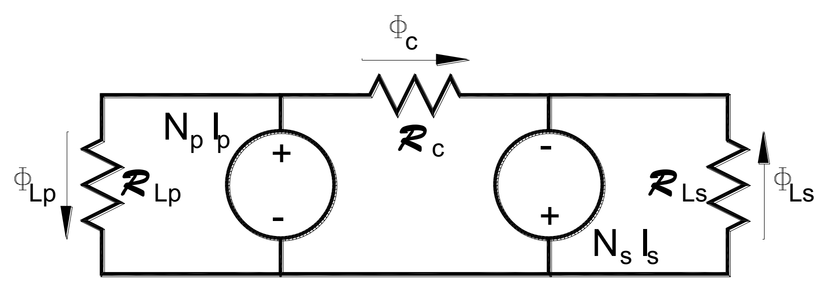
Apply Circuit Analogy to Transformer Model

- Introduce flux linkage
  - $\lambda = N\Phi$ (flux linked by N-turn loop, $V = \frac{d\lambda}{dt}$)
  - $\lambda_p = Np(\Phi_c + \Phi_{Lp}) = \frac{N_p^{\,2}}{R_c} I_p + \frac{N_p^{\,2}}{R_{Lp}} I_p + \frac{N_p N_s}{R_c} I_s$
  - $\lambda_s = Ns(\Phi_c + \Phi_{Ls}) = \frac{N_s^{\,2}}{R_c} I_s + \frac{N_s^{\,2}}{R_{Ls}} I_s + \frac{N_p N_s}{R_c} I_p$
- Two-port inductance matrix
  - $\begin{bmatrix} \lambda_p \\ \lambda_s \end{bmatrix} = \begin{bmatrix} \frac{N_p^{\,2}}{R_c} + \frac{N_p^{\,2}}{R_{Lp}} & \frac{N_p N_s}{R_c} \\ \frac{N_p N_s}{R_c} & \frac{N_s^{\,2}}{R_c} + \frac{N_s^{\,2}}{R_{Ls}} \end{bmatrix} \begin{bmatrix} I_p \\ I_s \end{bmatrix} = \begin{bmatrix} L_{11} & L_M \\ L_M & L_{22} \end{bmatrix} \begin{bmatrix} I_p \\ I_s \end{bmatrix}$
  - $\begin{bmatrix} V_p \\ V_s \end{bmatrix} = \begin{bmatrix} L_{11} & L_M \\ L_M & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_p \\ I_s \end{bmatrix}$
  - Where $L_{11} = L_{Lp} + L_c, \quad L_M = \frac{N_s}{N_p} L_c, \quad L_{22} = L_{Ls} + \left(\frac{N_s}{N_p}\right)^2 L_c$

Apply Circuit Analogy to Transformer Model

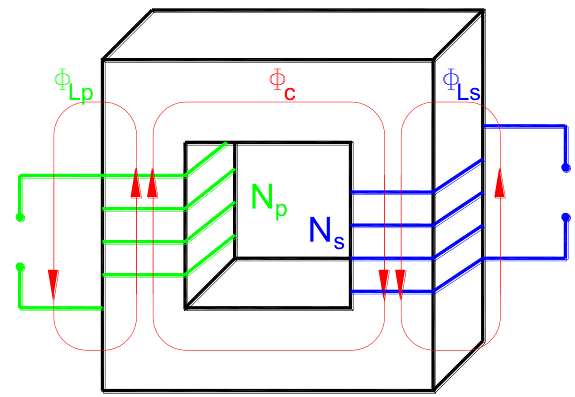
- Magnetizing (primary) inductance
  - $L_p = Lc = \frac{N_p}{N_s} L_M$
- Leakage inductances
  - Primary: $L_{Lp} = L_{11} - \frac{N_p}{N_s} L_M$
  - Secondary: $L_{Ls} = L_{22} - \frac{N_s}{N_p} L_M$
- Effective turns ratio
  - $n_e = \sqrt{\frac{L_{22}}{L_{11}}}$
- Coupling coefficient
  - $k = \frac{L_M}{\sqrt{L_{11} L_{22}}}$





Transformer Model: Practical Interpretation

- Primary (magnetizing) inductance
  - Open secondary, measure across primary
    - $L = L_p + L_{Lp}$
    - $L_p \gg L_{Lp}$, measure ~ L_p
  - Open primary, measure across secondary
    - $L = N^2 L_p + L_{Ls}$ (translate L to 2nd ary reference)
    - $N^2 L_p \gg L_{Ls}$, measure ~$N^2 L_p$



- Leakage inductance
  - Short secondary, measure across primary
    - $L = L_{Lp} + \frac{L_{Ls}}{N^2}$
    - Isolation transformer, N=1, $L_{Lp} \approx L_{Ls}$
    - HV step-up transformer, $L_{Lp} \ll L_{Ls}$ due to HV insulation on 2nd ary
    - $L_{Lp} = (1-k)L_p$

Transformer Applications

- Voltage/current scaling
  - HV generation
- Impedance matching
  - Few Ω < Z_{TL} < ~100 Ω
- Isolation
  - Floating ground
  - Block dc signal component

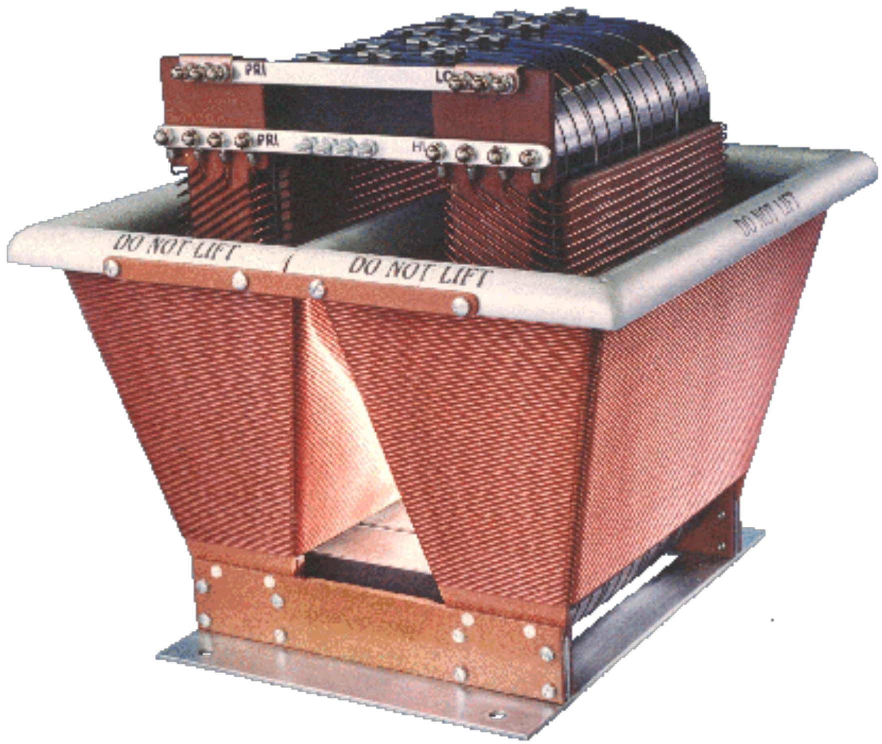
Transformer Types

SLAC

- AC
  - 60 Hz
  - Step-up (neon sign), step-down (filament), isolation
  - Decrease in size with increasing frequency
- Pulse
  - Uni-polar
  - Ubiquitous in low duty factor HV applications
- DC
  - Faraday's law, $\int V dt = NA \int dB$
  - $\int_0^{\infty} V dt \rightarrow \infty$ for DC, therefore,$NA\Delta B \rightarrow \infty$
  - Transformers get big as the frequency goes down

Pulse Transformers

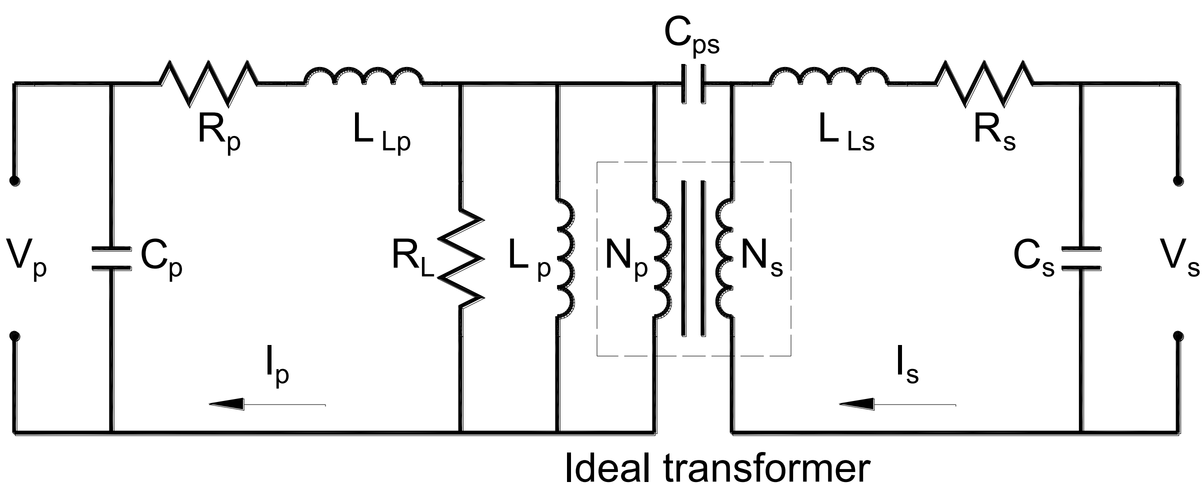
- Functions
  - Voltage gain
  - Impedance matching
  - Teach humility
- Commercial pulse transformers are generally made to order



Stangenes Industries klystron transformer

Transformers: Practical Limitations

- Core
  - Material limitations
    - $V\tau$ constraint
    - $\mu = f(\omega, I, \text{temperature})$
  - Typically gapped
- Primary inductance
  - $\tau_{droop} = L/R$
- Stray capacitance
  - Primary to secondary
    - In series with leakage inductance
    - Operate below self-resonance
  - Secondary inter-winding
    - Load secondary



Ideal transformer

Transmission Lines (TL)

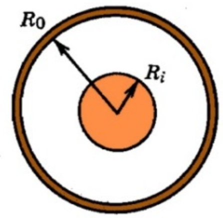
- Structure that provides a fixed impedance path for the propagation of electromagnetic energy
  - Coaxial cable
  - Rf waveguide
- Impedance
  - $Z = \sqrt{\frac{L}{C}}$
- Propagation velocity/time
  - $v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\left(\frac{\varepsilon}{\varepsilon_o}\right)\left(\frac{\mu}{\mu_o}\right)}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}}$
  - For a line of length, l, the propagation time/delay is
  - $\tau = \frac{l}{v} = l\sqrt{\varepsilon\mu} = \frac{l\sqrt{\varepsilon_r\mu_r}}{c}$

Common Transmission Line Geometries

TABLE 9.3 Properties of Common Transmission Lines, TEM Modes[a]

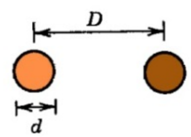
1 Coaxial transmission line:

$$\mathscr{C} = 2\pi\varepsilon/\ln(R_0/R_i)$$
$$\mathscr{L} = (\mu/2\pi)\ln(R_0/R_i)$$
$$Z_0 = (\sqrt{\mu/\varepsilon}/2\pi)\ln(R_0/R_i)$$



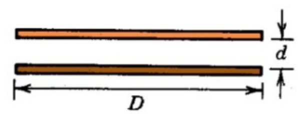
2 Two-wire transmission line

$$\mathscr{C} = \pi\varepsilon/\cosh^{-1}(D/d)$$
$$\mathscr{L} = \mu/\pi\,\cosh^{-1}(D/d)$$
$$Z_0 = (\sqrt{\mu/\varepsilon}/\pi)\cosh^{-1}(D/d)$$



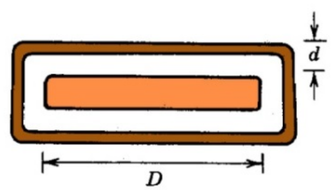
3 Isolated parallel plates ($d \ll D$)

$$\mathscr{C} = \varepsilon D/d$$
$$\mathscr{L} = \mu d/D$$
$$Z_0 = \sqrt{\mu/\varepsilon}(d/D)$$



4 Stripline ($d \ll D$)

$$\mathscr{C} = 2\varepsilon D/d$$
$$\mathscr{L} = \mu d/2D$$
$$Z_0 = \sqrt{\mu/\varepsilon}(d/2D)$$



[a] $\mathscr{C}$ = capacitance per unit length (farads/meter); $\mathscr{L}$ = inductance per unit length (henries/meter); Z_0 = characteristic impedance (ohms).

from "Principles of Charged Particle Acceleration," S. Humphries, Jr.

Discrete Element Transmission Line Approximation

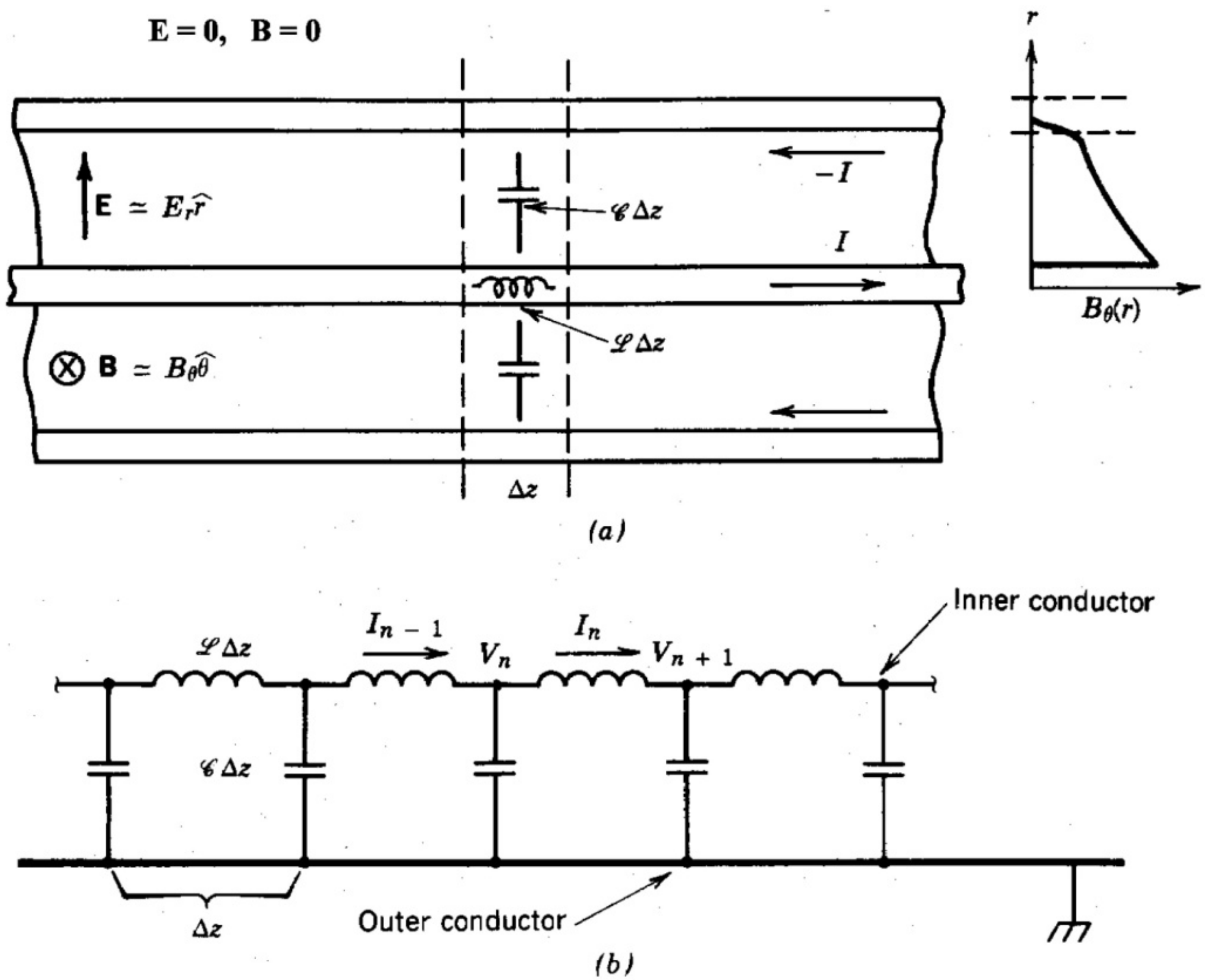
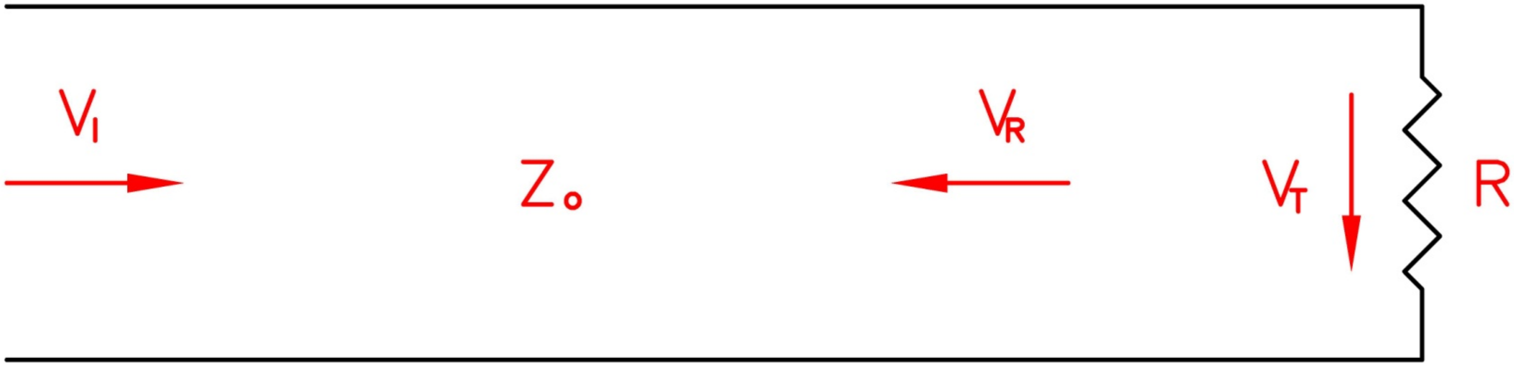


Figure 9.29 Coaxial transmission line. (*a*) Physical basis for lumped circuit element model of TEM wave propagation. (*b*) Lumped circuit element analog of a coaxial transmission line.

from, "Principles of Charged Particle Acceleration," S. Humphries, Jr.

Transmission Line Terminations

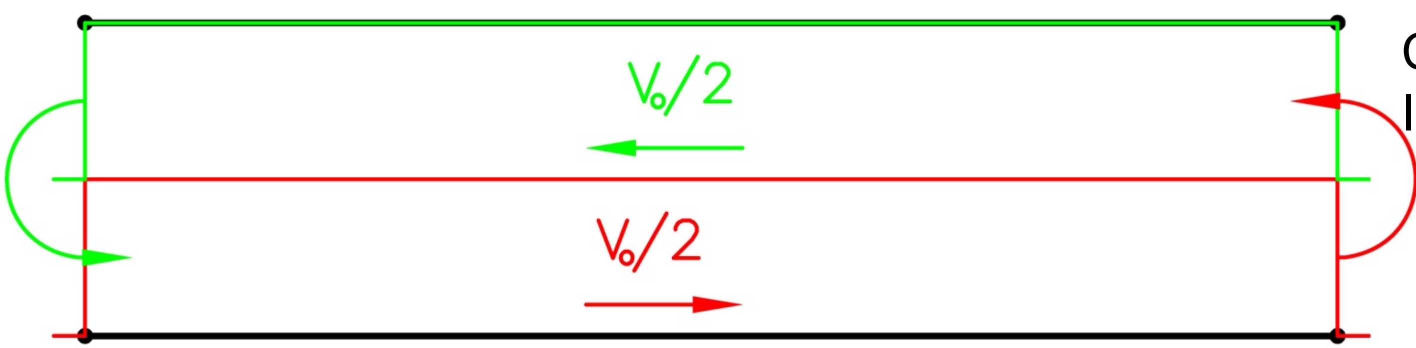


- Matched: $R = Z_O$, $V_T = V_I$, $V_R = 0$
- Open: $R = \infty$, $V_R = V_I$, $V_T = 2V_I$
- Short: $R = 0$, $V_R = -V_I$, $V_T = 0$
- General
  - Reflection coefficient $\Gamma = \frac{R-Z_o}{R+Z_o}$
  - Transmission coefficient $\mathrm{T} = \frac{2R}{R+Z_o}$
  - $\mathrm{T} - \Gamma = 1$
  - $V_T = \mathrm{T}V_I$
  - $V_R = \Gamma V_I$
  - $I_T = V_T/R = \mathrm{T}V_I/R$
  - $I_R = V_R/Z_o = \Gamma V_I/Z_o$

- V_I : Incident voltage
- V_R : Reflected voltage
- V_T : Transmitted voltage
- $V_I = V_T - V_R$
- I_I : Incident current = V_I/Z_O
- I_R : Reflected current
- I_T : Transmitted current
- $I_I = I_T + I_R$

Wave Model of Voltage Charged Transmission Line

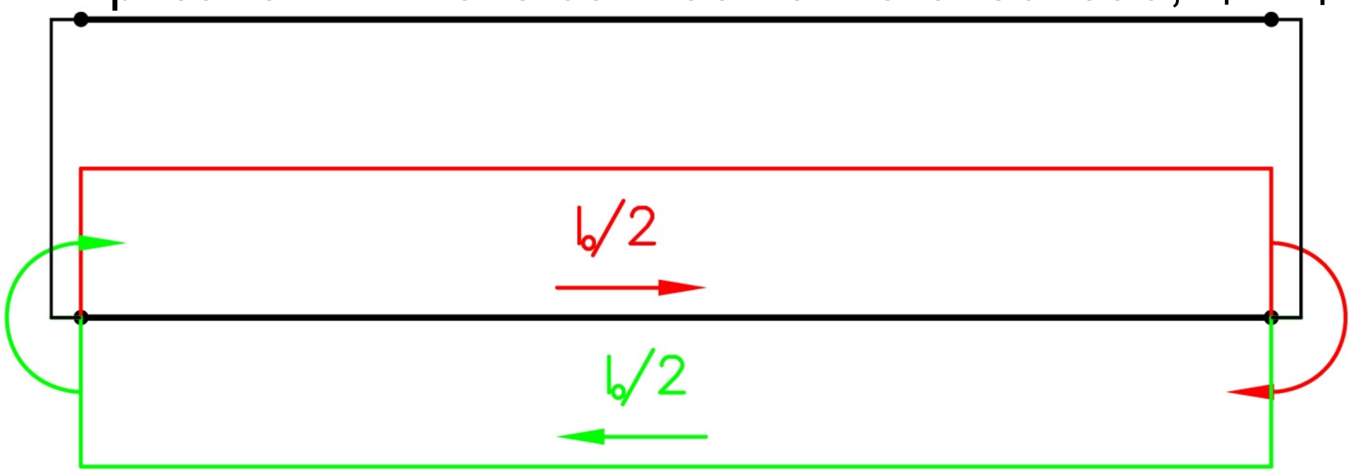
- Section of transmission charged to voltage, V_o, “open” at both ends
- Equivalent model
  - Propagating wave of voltage $V_o/2$ traveling left to right
  - Encounters open at end of line and reflects, same polarity and equal magnitude
  - Sum of left and right going waves is V_o
  - When left to right going wave reaches open at end it reflects and replenishes right to left going wave
- Implication: if line is connect to matched load, $V_T = V_I = V_o/2$



Charge voltage: V_o
Impedance: Z_o

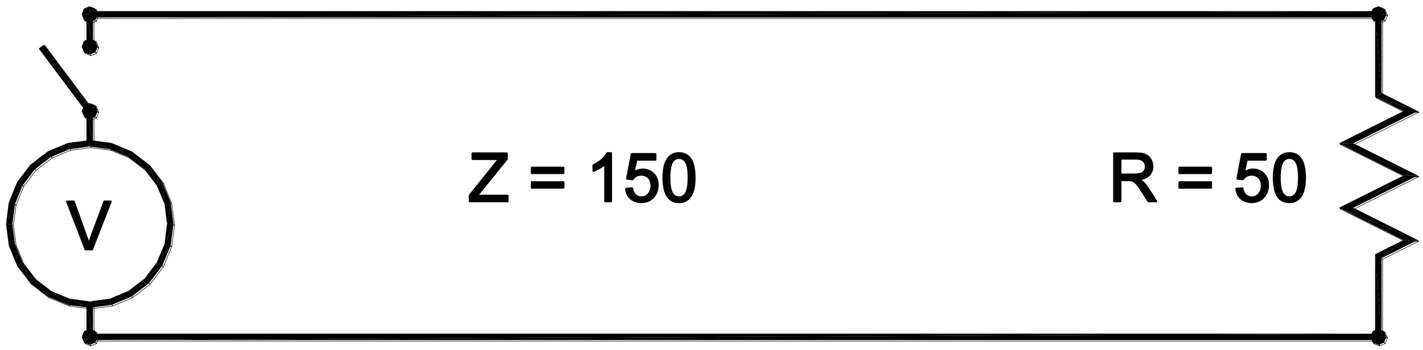
Wave Model of Current Charged Transmission Line

- Section of transmission charged to current, I_o, “shorted” at both ends
- Equivalent model
  - Propagating wave of current $I_o/2$ (and voltage $I_o Z_o/2$) traveling left to right
  - Encounters open at end of line and reflects, opposite polarity and equal magnitude
  - Sum of left and right going waves is $I = I_o$ and $V = 0$
  - When left to right going wave reaches short at end it reflects and replenishes right to left going wave
- Implication: if line is connect to matched load, $I_T = I_l = I_o/2$ and $V_T = I_o Z_o/2$



Charge current: I_o
Impedance: Z_o

Analysis of TL Step Response: Bounce Diagrams

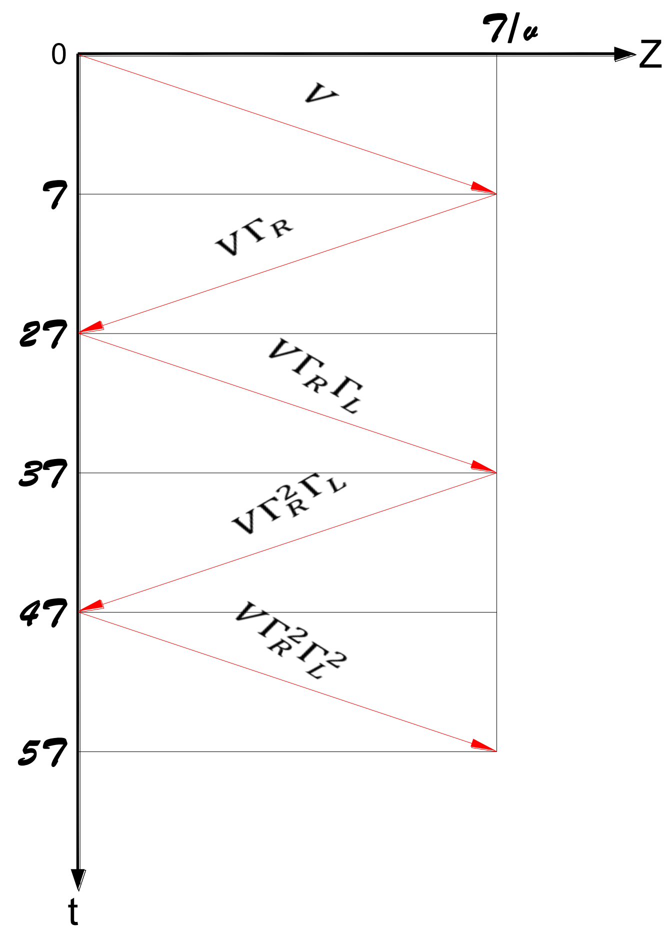


- $\Gamma_R = \frac{50-150}{50+150} = -\frac{1}{2}$
- $\Gamma_L = \frac{0-150}{0+150} = -1$

- At time, t=0, the switch closes and the perfect voltage source (Z=0) is applied to the transmission line of length, τ/v.
  - At t=0, a wave of voltage V starts to propagate down the TL
  - At t=τ the wave arrives at the mismatched load, some is reflected
  - At t=2τ the reflected wave arrives at the source, and is reflected
  -
  -
- Bounce diagrams are a useful tool to follow the time response

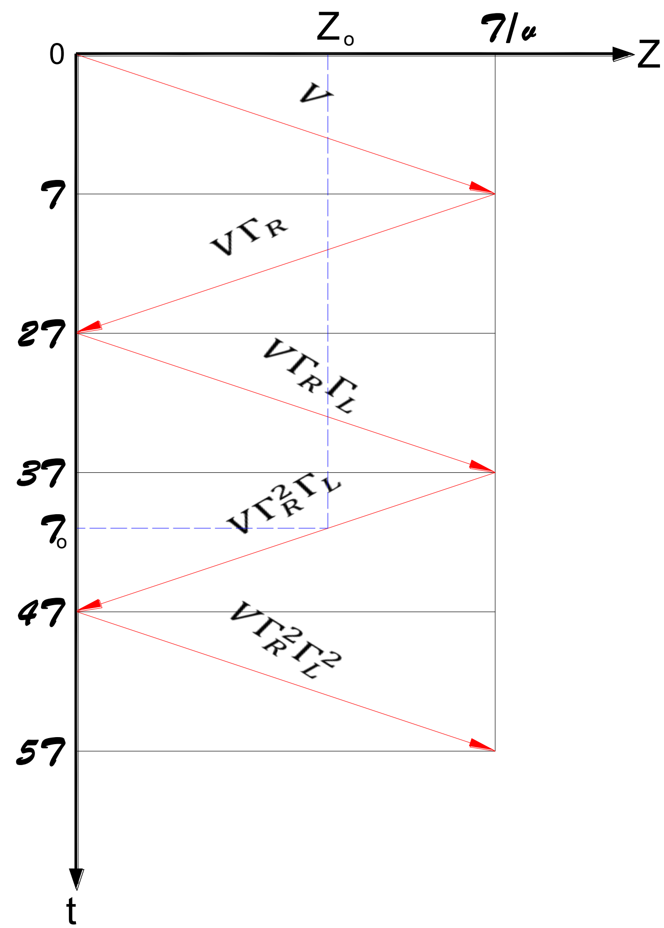
Bounce Diagram

- Horizontal axis: position
- Vertical axis: time
- Vectors: leading (or trailing) edge of TL waves, labeled with voltage (current) amplitude
  - Amplitude is product of incident wave and reflection coefficient at discontinuity
- Determine voltage (current)
  - As a function of position, at a specific time
  - As a function of time, at a specific position



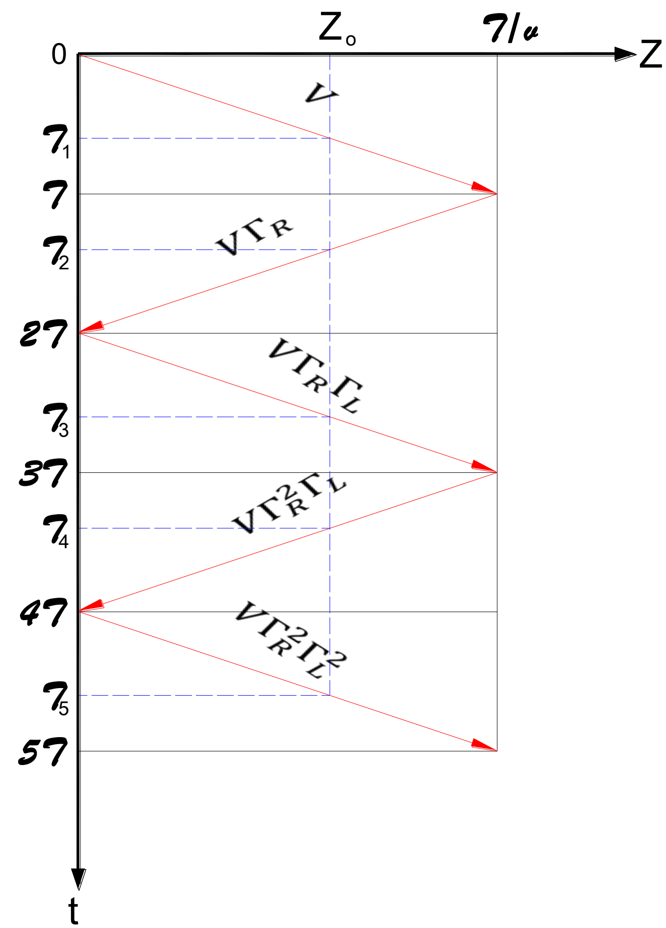
Bounce Diagram Interpretation: $t = \tau_o$

- Mark τ_o on the time axis
- Draw a horizontal line at τ_o
- Draw a vertical line from the intersection of the τ_o line and the wave vector, position z_o is the location of the leading edge of the wave
- Voltage along TL to the left of z_o is the sum of all wave vectors intersecting an imaginary line at z_o^-, and to the right it is the sum of the vectors intersected at z_o^+
  - $V(z < z_0, \tau_0) = V + V\Gamma_R + V\Gamma_R\Gamma_L = V(1 - \frac{1}{2} + \frac{1}{2})$
  - $V(z < z_0, \tau_0) = V + V\Gamma_R + V\Gamma_R\Gamma_L + V\Gamma_R^2\Gamma_L = V(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{4})$



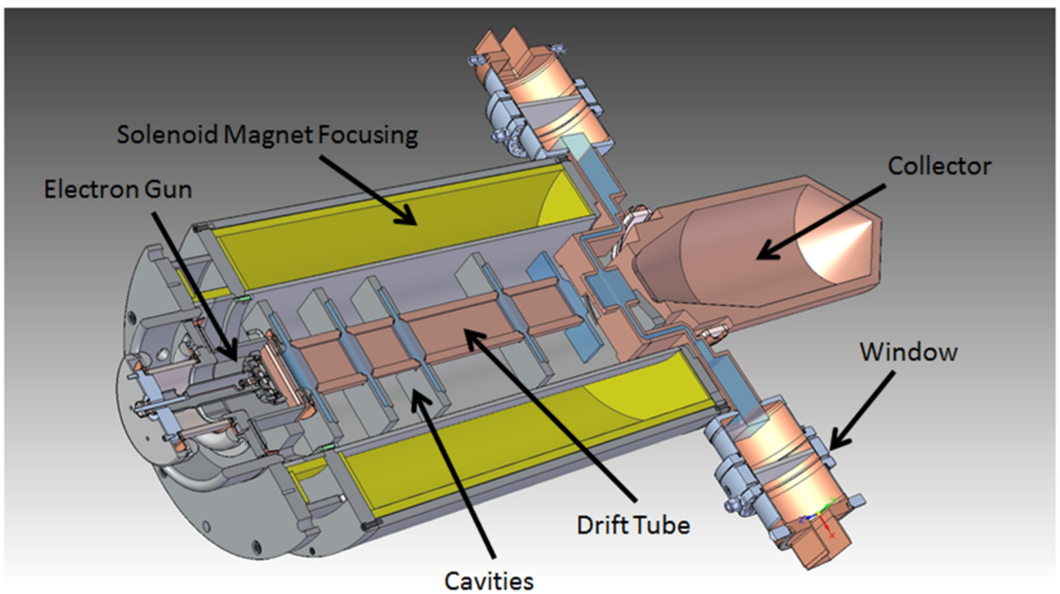
Bounce Diagram Interpretation: $z = z_o$

- Mark z_o on the position axis
- Draw a vertical line at z_o
- Draw a horizontal line at each intersection of the z_o line and a wave vector. Each τ_N is the time when the new wave fronts arrive at z_o
- Voltage at z_o versus time is then:
  - $0 \rightarrow \tau_1^-$ $\quad 0$
  - $\tau_1^+ \rightarrow \tau_2^-$ $\quad V$
  - $\tau_2^+ \rightarrow \tau_3^-$ $\quad V + V\Gamma_R$
  - $\tau_3^+ \rightarrow \tau_4^-$ $\quad V + V\Gamma_R + V\Gamma_R\Gamma_L$
  - $\tau_4^+ \rightarrow \tau_5^-$ $\quad V + V\Gamma_R + V\Gamma_R\Gamma_L + V\Gamma_R^2\Gamma_L$
  - $\tau_5^+ \rightarrow \tau_6^-$ $\quad V + V\Gamma_R + V\Gamma_R\Gamma_L + V\Gamma_R^2\Gamma_L + V\Gamma_R^2\Gamma_L^2$
- The TL voltage $\rightarrow V$ as the wave transients "damp out" over several transit times



Klystrons

- Purpose: convert low frequency electrical power to radio frequency EM power
- Capable of producing very high peak RF power, up to ~100 MW, with a nearly constant phase and amplitude for the bulk of the output pulse



Klystron Load Constraints

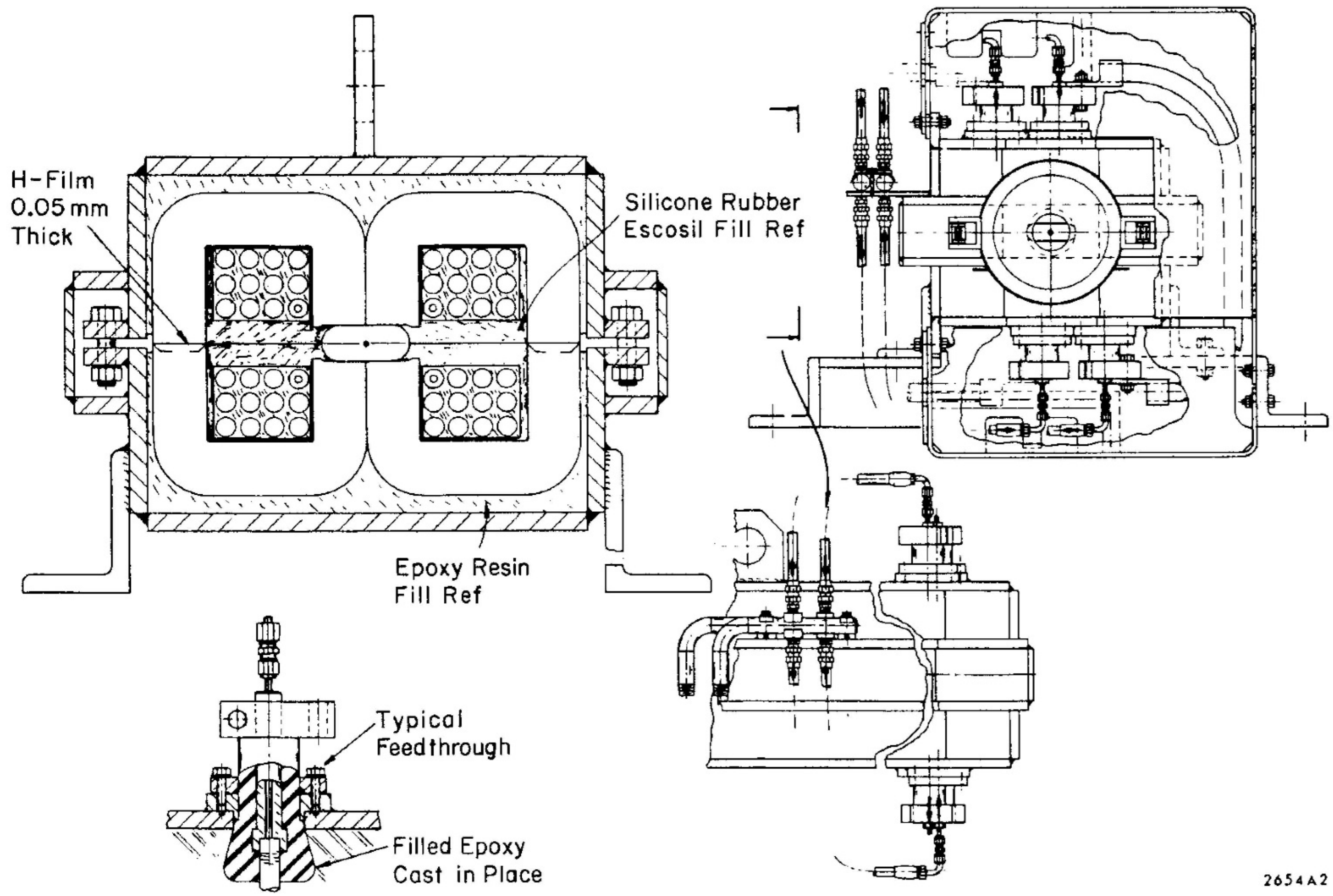
- Amplifiers: output regulation limited by input regulation
  - Low level RF (LLRF)
  - Beam acceleration voltage
    - RF phase α beam voltage
    - 0.1º phase stability typically required
    - Necessitates beam voltage stability to <50 V on >100 kV, <500 ppm
      - LCLS critical stations require ~30 ppm
  - Beam focusing fields (typically solenoid current)
- Electron beam device, operates with space-charge limited emission
  - $I_{beam} = \mu V^{1.5}$
  - Perveance, μ, typically $\sim 10^{-6}$
  - $Z = V/I = 1/ \mu V^{0.5}$
  - $P_{beam} = VI = \mu V^{2.5} = P_{RF} /0.5$ (typical, RF efficiency range 30 – 70%)

Beam Kickers

- Purpose: selectively deflect a portion of a charged particle beam into an alternative transport channel
- Two general types
  - Lumped inductance
    - Kicker is an electromagnet
    - Beam deflected by magnetic field
    - High current modulator
  - Transmission line
    - Kicker presents a fixed impedance to the modulator
    - Terminated into a matched impedance to avoid reflections
    - Typically uses both E and B to deflect beam
    - No intrinsic rise/fall time, can be used in systems with small inter-bunch spacing

Lumped Inductance Kicker

- Full sine
- Half sine



Transmission Liner Kicker

- Vacuum
  - Higher frequency fidelity
- Lumped element
  - Greater deflection field